CONTROLLER DESIGN OF CHUAS CIRCUIT BY SLIDING MODE CONTROL AND LMI

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Submission date: 07-Feb-2019 12:45AM (UTC-0500)
Submission ID: 1074346694
File name: LLER_DESIGN_OF_CHUAS_CIRCUIT_BY_SLIDING_MODE_CONTROL_AND_LMI.pdf (153.7K)
Word count: 2513
Character count: 10798

Proceedings of National Seminar on Applied Technology, Science, and Arts (1st APTECS), Surabaya, 22 Dec. 2009, ISSN 2086-1931

Controller Design of Chua's Circuit by Sliding Mode Control and LMI

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Abstract **In this paper, we studied controller design of Chua's circuit which is chaos system. This system contents the non linear part and we applied the sliding mode control as design controller. Here, it is given the dynamic compensator to avoid the chattering in the sliding mode control. The dynamical compensator is design by using the linear matrix inequality (LMI). The simulation has been done by Matlab. Some of dynamic compensator is tried to observe the performance of system.**

Keywords **Chaos control, sliding mode control, dynamic compensator, LMI**

1. Introduction

The study of chaos system has been done by some researchers. The chaos is happen because the system has certain parameter, and if the parameter has little changes then the system performance is changing and uncontrollable For long time ago, it is stated that the chaos is unpredictable and uncontrollable. But in 1990, Ott did research in the chaos system [1], and after that a lot of research are doing in various fields. Chua's circuit and Lorenz system are chaos systems. In 2007, [2] studied the chaos synchronization by control based on the three diagonal structure, and [2] studied the synchronize method of fractional Lorenz system, Chen system and Chua's circuit.

Sliding mode control is one of the controller design method which is used in a lot of area. The sliding mode controller is applied in the chaos system [1]. The sliding mode controller is adopted for the robust design controller of uncertain system, because the sliding mode controller is easy o apply, has a quick response, has good transient performance and insensitivity in plant parameter and external disturbance [1].

Usually, the process has two steps, those are design Of sliding surface to assure that the sliding mode equation is stable, and the second is the controller design to drive the state system to sliding surface in finite time. In recent year, variable structure design technique of sliding mode is a popular method in chaos system. Such as [3] studied the chaos control by using sliding, mode theory, and also [4].

In sliding mode controller usually the chattering is happen, the chattering disturbs the performance of system. [1] proposed the new reached law and derived the continue controller to reduce the chattering phenomena. The parameter of sliding surface is obtained by solving the linear matrix inequality (LMI). To handle the linearity part, [1] give the dynamic compensator such that the stability of system is increased.

In this paper, we applied the sliding mode controller to Chua's Circuit, we derive the feedback control, the LMI by using the Lyapunov stability theorem. We choose some value of dynamic compensator and sliding coefficient such that the LMI is satisfied. Some simulation are made by Matlab program. The performance of Chua's circuit system is observed.

2. Mathematical Model of Chua's Circuit.

The Chua's Circuit is a simple electronic circuit made of two capacitors, one linear resistor, one inductor ad one non linear diode [5]. Suppose x_1, x_2, x_3 is a current through the inductor, a voltage across capacitor C_1 and across the inductor and the voltage across the capacitor C_2, respectively then system can be written as mathematical model [1]:

$$\begin{aligned} \dot{x}_1(t) &= \quad x_1 + x_2 + x_3 \\ \dot{x}_2(t) &= \quad x_1 + u_1 \\ \dot{x}_3(t) &= \quad (x_1 \quad x_3 \quad g(x_3)) + u_2 \end{aligned} \tag{1}$$

with non linear part

$$g(x_3) = nx_3 + \frac{1}{2}(m \quad n)(|x_3 + 1| \quad |x_3 \quad 1|)$$

The chaos will be happen if the value of parameters are

$$= 40, \quad = 93.333, m = \quad 1.139, n = \quad 0.711$$

and $u_1 = u_2 = 0$.

The system Eq (1) can be written as state space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \quad 1 & 1 & 1 \\ & 0 & 0 \\ & 0 & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g(x_3) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2}$$

And we can write as general equation

$$\dot{x}(t) = Ax + f(x) + Bu \tag{3}$$

The system (2) and (3) is unstable. Now, we design the controller such that those system are stable. Here we use the sliding mode controller as design control and we observe the system performance.

3. Sliding Mode Controller

The sliding mode controller method, usually has two steps. The first one is the design sliding surface and the second is the design controller such that the states of system reach the sliding surface.

3.1. Sliding Surface Design

Suppose the sliding surface is

$$s = Cx + z \tag{4}$$

where $C, s, x, z,$ are matrix which is determined, sliding surface, state variable, the state dynamic compensator, respectively. The state dynamic compensator satisfied:

$$\dot{z} = Kx \quad z \tag{5}$$

Matrix K will be determined by solving the linear matrix inequality (LMI).

If we take the differential of sliding surface in Eq (5) respect to system Eq (3) we obtained

$$\dot{s} = C\dot{x} + \dot{z} = CAx + Cf(x) + CBu + Kx \quad z \tag{6}$$

In sliding surface, $\dot{s} = 0$ so that we get the control equivalent

$$u_{eq} = (CB)^{\ 1}(\ CAx \quad Cf(x) \quad Kx + z) \tag{7}$$

The matrix C will be determined and satisfies $CB = I$ such that control equivalent become

$$u_{eq} = \quad CAx \quad Cf(x) \quad Kx + z \tag{8}$$

We substitute the equation (8) to equation (3) then we get

$$\begin{aligned} \dot{x} &= Ax + f(x) + B(\ CAx \quad Cf(x) \quad Kx + z) \\ &= \{A \quad BCA \quad BK\}x + Bz \end{aligned} \tag{9}$$

In sliding surface, we know that $s = 0$ so that $z = \quad Cx$ and equation (9) can be written as

$$\begin{aligned} \dot{x} &= \{A \quad BCA \quad BK\}x \quad BCx \\ &= \{A \quad BCA \quad BK \quad BC\}x \\ &= \{A \quad B(K + C + CA)\}x \end{aligned} \tag{10}$$

Suppose matrix, $M = \quad (K + C + CA)$ then the equation (10) can be written as

$$\dot{x} = (A + BM)x \tag{11}$$

According the control theory, if (A,B) is controllable then there is always exist the matrix M such that matrix $A + BM$ negative definite ($A + BM < 0$). And the eigen value of matrix $A + BM$ can be place in half left plane or have a negative real part. Therefore, we can adopt the pole assignment method to obtain matrices K,C . The matrices K and C can be obtained by solving the linear matrix inequality (LMI). The LMI is derived from the Lyapunov stability criteria.

Suppose a vector $V(x) = x^T x$ is a Lyapunov function candidate of equation (11). The system (11) is stable if satisfies:

a. $V(x,t) \quad 0, \ V(0,t) = 0$

b. $\dot{V}(x,t) < 0$

We know that $V(x) = x^T x > 0$ and

$$\dot{V} = \dot{x}^T x + x^T \dot{x} \tag{12}$$

Substitute from equation (11) to equation (12) we get

$\dot{V} = (A \quad B(K + C + CA))x)^T x$

$+ x^T (A \quad B(K + C + CA))x$

or

$$\dot{V} = x^T (A^T \quad K^T B^T \quad C^T B^T \quad A^T C^T B^T + A \quad BK \quad BC \quad BCA)x \qquad (13)$$

Suppose

$$= A^T \quad K^T B^T \quad C^T B^T \quad A^T C^T B^T + A \quad BK \quad BC \quad BCA$$

Then we can write equation (13) as $\dot{V} = x^T \quad x$.

If $\quad < 0$ (negative definite) then $\dot{V} < 0$ that is mean the system (11) asymptotical stable. The inequality

$$A^T \quad K^T B^T \quad C^T B^T \quad A^T C^T B^T + A \quad BK \quad BC \quad BCA < 0 \qquad (14)$$

is known as linear matrix inequality (LMI).

Now, we must determine matrices C and K such that the matrix inequality $\quad < 0$ satisfies.

In the Chua's circuit equation (2) we have matrices

$$A = \begin{bmatrix} & 1 & 1 \\ & 0 & 0 \\ & 0 & \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} C_{11} & 1 & 0 \\ C_{21} & 0 & 1 \end{bmatrix}, K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix},$$

So that we get

$$= \begin{bmatrix} 2 & 1 \quad k_{11} & 1 + k_{21} \\ k_{11} + 1 & 2k_{12} \quad 2C_{11} \quad 2 & k_{13} \quad C_{11} \quad k_{22} \quad C_{21} \\ k_{21} + 1 & k_{22} \quad C_{21} \quad k_{13} \quad C_{11} & 2k_{23} \quad 2C_{21} \quad 2 \end{bmatrix}$$

By Liapunov stability criteria, if matrix $\quad$ negative definite then the system (11) asymptotical stable. The value of matrices C and K such that matrix negative definite that is mean we solve the linear matrix inequality-LMI equation (14).

3.2. Design Controller

According [1], suppose we take the reached law

$$\dot{s}(t) = (\quad + \quad \| s \| \quad ^{1})s, 0 < \quad < 1, \quad , \quad > 0$$

Then the state variable $x(t)$ will reach the sliding surface $\{x \mid s = 0\}$ in finite time, and by applying the control vector

$$u = \quad CAx \quad Cf \quad Kx + z \quad (\quad + \quad \| s \| \quad ^{1})s \qquad (15)$$

Then the state of system (11) from arbitrary initial condition will toward to sliding switching and reach the sliding surface in finite time [1].

So, the system (2) or (3) will be asymptotical stable if we apply the control input (15) with value of matrices C and K satisfies equation (14).

4. Simulation Result

In this simulation we take the parameter as follows [5] $= 40, \quad = 93.333, m = \quad 1.139, n = \quad 0.711$, the initial state $(x_1(0), x_2(0), x_3(0)) = (1, 0, \quad 1)$ and the initial compensator state $(z_1(0), z_2(0)) = (1, \quad 2)$, parameter $= 1, \quad = 2, \quad = \frac{1}{2}$. Here we take some values of C and K which is satisfies equation (14). For case 1

$$C = \begin{bmatrix} 31 & 1 & 0 \\ 40 & 0 & 1 \end{bmatrix}; K = \begin{bmatrix} 1 & 30 & 30 \\ 1 & 2 & 50 \end{bmatrix}$$

The inequality (14) is satisfies and the system asymptotical stable. Figure 1a. shows the performance of Chua's circuit without control (u=0). The system (2) is unstable, but after we applied the sliding mode control with dynamical compensator, the system is asymptotical stable (Fig. 1b.). Figure (1c) shows the Chua's circuit system with control in three dimensional. The control input is showed in Figure (1d).

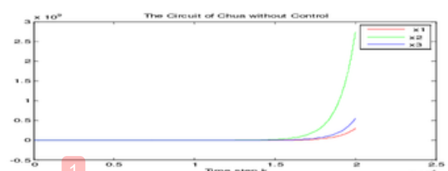

Figure 1a. The system without control

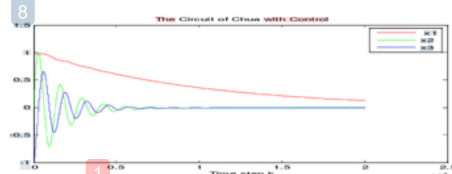

Figure 1b. The system with control

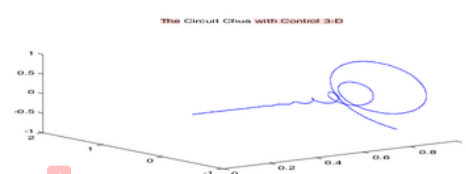

Figure 1c. The system with control 3-D

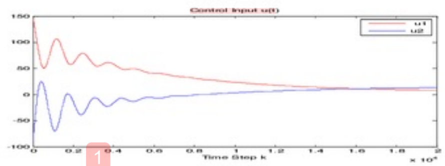

Figure 1d. Control input u(t)

For case 2:

$$C = \begin{bmatrix} 3 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}; K = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 5 \end{bmatrix}$$

The performance Chua's circuit with control is showed Figure 2a. The system with control in three dimension is showed Figure 2b. and the control input vector is showed Figure 2c.

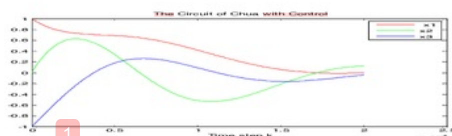

Figure 2a. The system with control

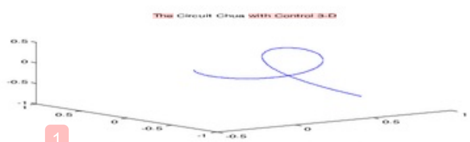

Figure 2b. The system with control 3-D

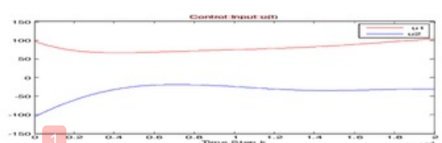

Figure 2c. The control input vector u(t)

For those case 2, the performance is go to stable condition slower than the case 1.

For case 3:

$$C = \begin{bmatrix} 71.4856 & 1 & 0 \\ 137.935 & 0 & 1 \end{bmatrix};$$

$$K = \begin{bmatrix} 1 & 70.9894 & 33.2247 \\ 1 & 3.2247 & 138.4314 \end{bmatrix}$$

The value of C and K are taken from [1]. For this case, Figure 3a. shows the Chua's circuit with control. System (2) converge to zeros, with an oscillation. Case 1 and Case 2 have less oscillation than case 3. The Figure 3b. is a system state in three dimension, and the Figure 3c. shows the control input u(t). This input has a sinusoidal form.

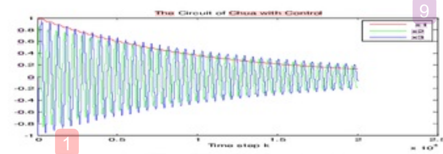

Figure 3a. The Chua's circuit with control

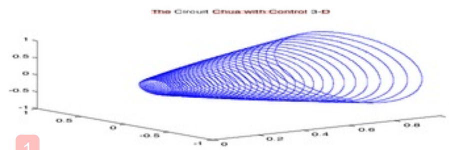

Figure 3b. The Chua's circuit with control in 3-D

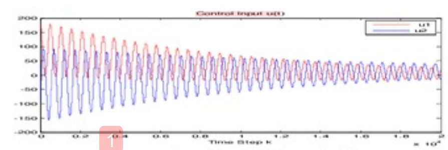

Figure 3c. Control input u(t)

From those simulation, we know that the value of C and K are influenced the stability of system. The control input u(t) also influences the system behavior. If the input control with sinusoidal form, give the behavior system is also has the sinusoidal form. The performance of case 1 is the best of three cases, because the system toward the stability condition with less oscillation.

5. Concluding Remark

From those discussion and the simulation, we conclude that:

a. The sliding mode control with dynamical compensator can be used as controller design for non linear system the Chua's circuit.
b. The value of sliding coefficient C and the dynamic compensator K can be determined by solving the LMI which is derived from the Liapunov stability criteria.
c. The values of C and K are influence the speed of the system become stable
d. The performance of case 1 is better than the others. Because case 1 has less oscillation, and faster to go stable position.

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